Exercise 7

Solve the differential equation.

$$3y'' = 4y'$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad y' = re^{rx} \quad \rightarrow \quad y'' = r^2e^{rx}$$

Plug these formulas into the ODE.

$$3(r^2e^{rx}) = 4(re^{rx})$$

Divide both sides by e^{rx}.

$$3r^2 = 4r$$

Solve for r.

$$3r^2 - 4r = 0$$

$$r(3r - 4) = 0$$

$$r = \left\{0, \frac{4}{3}\right\}$$

Two solutions to the ODE are $e^0 = 1$ and $e^{4x/3}$. By the principle of superposition, then,

$$y(x) = C_1 + C_2e^{4x/3},$$

where C_1 and C_2 are arbitrary constants.